MASTER COACHING

NAME

PYTHAGORAS THEOREM

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YEAR 7

Edition 1

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Pythagoras' Theorem $A^2 + B^2 = C^2$

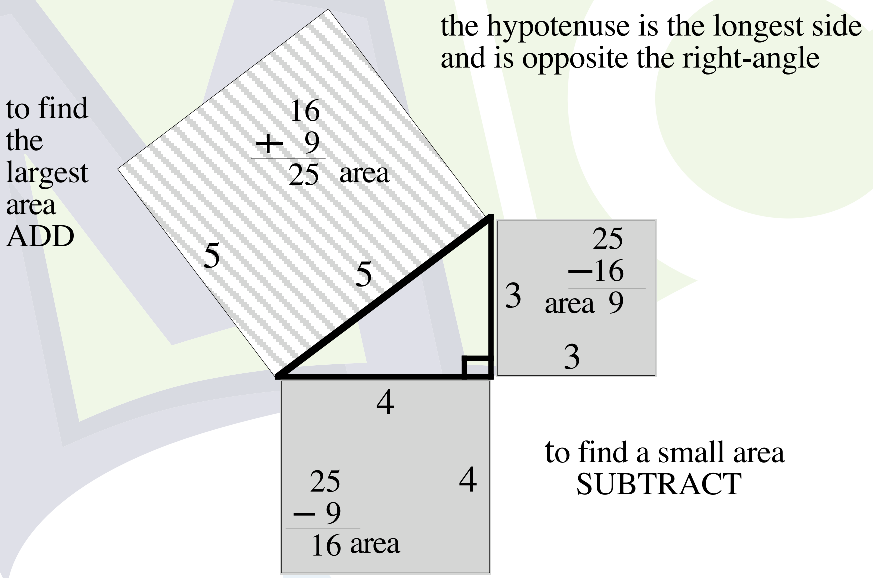
Most students learn $a^2 + b^2 = c^2$ without understanding the geometrical concepts involved.
It is *easier* to understand this topic from a geometrical sense without resorting to algebra.

We are talking in fact about three <u>areas</u> when we state :

> *"The square on the hypotenuse equals the sum of the squares on the other two sides."*

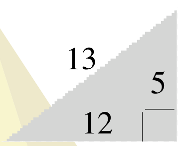
Note that this theorem needs triangles that have a **right angle** ... there must be a $90°$ angle !

Draw a diagram and actually write down the areas.
The two smaller squares added together equal the size of the big one
Your eyes will help explain the concept of Pythagoras' theorem.



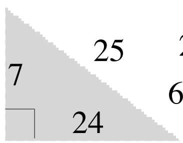
If all three sides are *whole* numbers they are said to be a **Pythagorean triad**.

eg



$13^2 = 5^2 + 12^2$

$169 = 25 + 144$



$25^2 = 24^2 + 7^2$

$625 = 576 + 49$

Some more of the infinite number of different Pythagorean triads are :
6 , 8 , 10   8 , 15 , 17   20 , 21 , 29   33 , 56 , 65

Pythagoras was the son of a Greek merchant. Born 570 BC , he died 501 BC
He probably learned this triangle fact while in Egypt ... he was able to *prove* it was true.
He taught the 'secret knowledge' at his school in a Greek colony at the foot of modern Italy.

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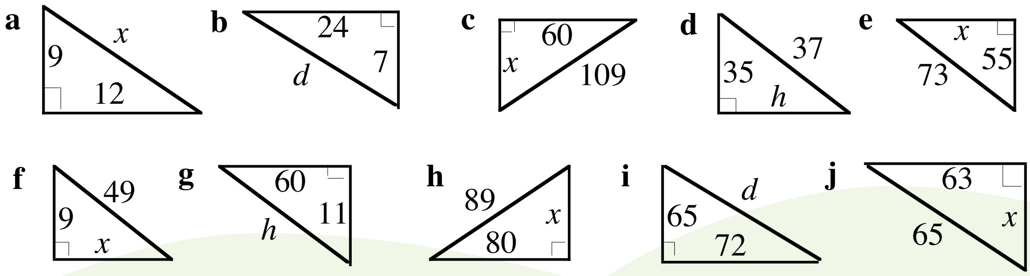
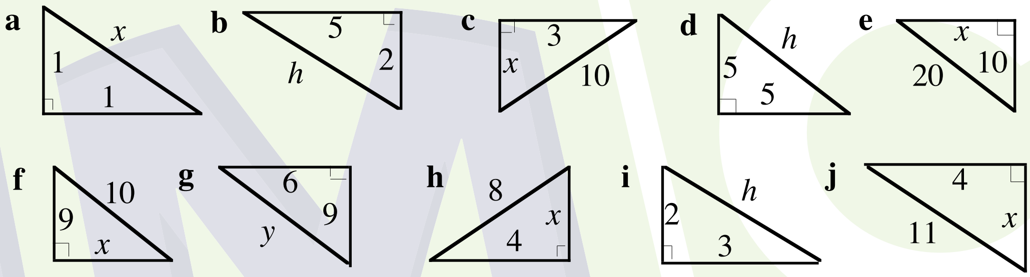
Exercises : **Pythagoras' Theorem**

Note that these diagrams are <u>not</u> drawn to scale.

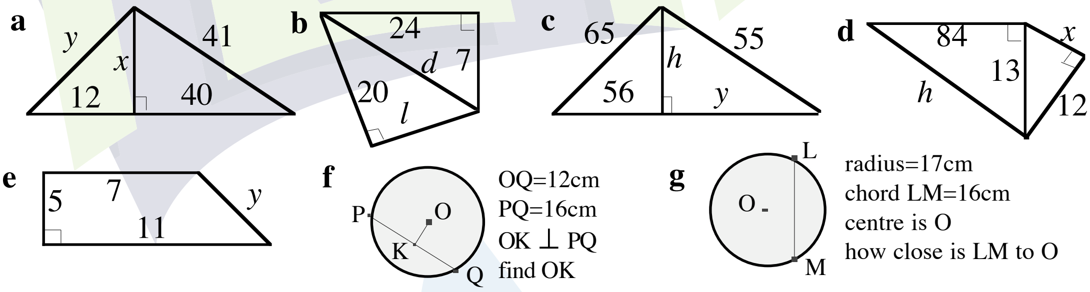
1 Find the value of the pronumerals in these diagrams. All these answers are *integers*.



2 Find the value of the pronumerals in these diagrams. All these answers are *surds*.



3 Find the value of the pronumerals in these diagrams.



4 **a** What is the longest thin needle that can fit inside a box 12cm by 6cm by 4cm?

b Clarisse hikes 8km east and then 5km south. How far is she from her starting place?

c A rhombus of side17cm has a long diagonal of 30cm. How long is the short diagonal?

d A bee goes 9m west, then 4m north, then up 3m. How far is it from its starting place?

Answers ... you can simplify some of these surds ...

1 **a** 15 **b** 25 **c** 91 **d** 12 **e** 48 **f** 40 **g** 61 **h** 39 **i** 97 **j** 16

2 **a** $\sqrt{2}$ **b** $\sqrt{29}$ **c** $\sqrt{91}$ **d** $\sqrt{50}$ **e** $\sqrt{300}$ **f** $\sqrt{19}$ **g** $\sqrt{117}$ **h** $\sqrt{48}$ **i** $\sqrt{13}$ **j** $\sqrt{105}$

3 **a** 9,15 **b** 25,15 **c** 33,44 **d** 85,5 **e** $\sqrt{41}$ **f** $\sqrt{80}$ cm **g** 5cm

4 **a** 14cm **b** $\sqrt{89}$ km **c** 16cm **d** $\sqrt{106}$ m

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Surds : see separate set of notes in this series ... " Master Coaching Surds "

Multiplying × and dividing ÷ are strong operations and can 'break' the square root sign :

$\sqrt{\mathbf{A} \times \mathbf{B}} = \sqrt{\mathbf{A}} \times \sqrt{\mathbf{B}}$ $\quad \sqrt{21} = \sqrt{3 \times 7} = \sqrt{3} \times \sqrt{7}$ $\quad \sqrt{48} = \sqrt{6 \times 8} = \sqrt{2} \times \sqrt{24} = \sqrt{3} \times \sqrt{16}$

$\sqrt{\mathbf{A}} \times \sqrt{\mathbf{B}} = \sqrt{\mathbf{A} \times \mathbf{B}}$ $\quad \sqrt{4} \times \sqrt{5} = \sqrt{20}$ $\quad \sqrt{7} \times \sqrt{8} = \sqrt{56}$ $\quad \sqrt{5} \times \sqrt{5} = \sqrt{25}$

$\sqrt{\mathbf{A} \div \mathbf{B}} = \sqrt{\mathbf{A}} \div \sqrt{\mathbf{B}}$ $\quad \sqrt{34 \div 2} = \sqrt{34} \div \sqrt{2}$ $\quad \sqrt{18 \div 3} = \sqrt{18} \div \sqrt{3}$

$\sqrt{\frac{\mathbf{A}}{\mathbf{B}}} = \frac{\sqrt{\mathbf{A}}}{\sqrt{\mathbf{B}}}$ $\quad \sqrt{\frac{28}{7}} = \frac{\sqrt{28}}{\sqrt{7}}$ $\quad \sqrt{\frac{55}{11}} = \frac{\sqrt{55}}{\sqrt{11}}$ $\quad \sqrt{\frac{91}{13}} = \frac{\sqrt{91}}{\sqrt{13}}$

Addition × and Subtraction ÷ are weak, and <u>can not</u> 'break' the square root sign :

$\sqrt{A + B} \neq \sqrt{A} + \sqrt{B}$ $\quad \sqrt{16 + 9} \neq \sqrt{16} + \sqrt{9}$ $\quad \sqrt{7^2 + 3^2} \neq 7 + 3$

$\sqrt{A - B} \neq \sqrt{A} - \sqrt{B}$ $\quad \sqrt{16 - 9} \neq 4 - 3$ $\quad \sqrt{6^2 - 2^2} \neq 6 - 2$

By definition $\sqrt{\mathbf{A}^2} = \mathbf{A}$, and of course $\left(\sqrt{\mathbf{A}}\right)^2 = \mathbf{A}$ ie $\sqrt{\mathbf{A}} \times \sqrt{\mathbf{A}} = \mathbf{A}$

$\sqrt{7^2} = 7$, and of course $\left(\sqrt{7}\right)^2 = 7$ ie $\sqrt{7} \times \sqrt{7} = \sqrt{49} = 7$

Factorising ... keep the same power :

$10^2 = 2^2 \times 5^2$ $\quad 10^3 = 2^3 \times 5^3$ $\quad 12^7 = 3^7 \times 4^7$ $\quad 15^{11} = 3^{11} \times 5^{11}$ $\quad 28^5 = 4^5 \times 7^5$

$21^2 + 35^2 = 7^2(3^2 + 5^2)$ $\quad 8^2 + 6^2 = 2^2(4^2 + 3^2)$ $\quad 22^2 + 33^2 = 11^2(2^2 + 3^2)$

$15^2 + 12^2 = 3^2(5^2 + 4^2)$ $\quad 14^8 + 21^8 = 7^8(2^8 + 3^8)$ $\quad 27^2 + 63^2 = 9^2(3^2 + 7^2)$

In Pythagoras questions (and solving quadratic equations) the power will only be 2.

Pythagoras ... calculations can sometimes be made simpler by the following logic :

$h^2 = 6^2 + 10^2$	$d^2 = 25^2 + 20^2$	$l^2 = 28^2 - 28^2$	$x^2 = 77^2 - 66^2$
$h^2 = 2^2 \times (3^2 + 5^2)$	$d^2 = 5^2 \times (5^2 + 4^2)$	$l^2 = 7^2 \times (4^2 + 3^2)$	$x^2 = 11^2 \times (7^2 - 6^2)$
$h = \sqrt{2^2 \times (3^2 + 5^2)}$	$d = \sqrt{5^2 \times (5^2 + 4^2)}$	$l = \sqrt{7^2 \times (4^2 + 3^2)}$	$x = \sqrt{11^2 \times (7^2 - 6^2)}$
$h = \sqrt{2^2} \times \sqrt{3^2 + 5^2}$	$d = \sqrt{5^2} \times \sqrt{5^2 + 4^2}$	$l = \sqrt{7^2} \times \sqrt{4^2 + 3^2}$	$x = \sqrt{11^2} \times \sqrt{7^2 - 6^2}$
$h = 2 \times \sqrt{9 + 25}$	$d = 5 \times \sqrt{25 + 16}$	$l = 7 \times \sqrt{16 - 9}$	$x = 11 \times \sqrt{49 - 36}$
$h = 2\sqrt{34}$	$d = 5\sqrt{41}$	$l = 7\sqrt{7}$	$x = 11\sqrt{13}$

Of course with a bit of practice you can do *all* of this in your head!
Can you visualise the steps in these :

$h^2 = 12^2 + 18^2$	$d^2 = 17^2 + 34^2$	$l^2 = 70^2 - 40^2$	$x^2 = 54^2 - 45^2$
$h =$	$d =$	$l =$	$x =$
$h = 6\sqrt{13}$	$d = 17\sqrt{5}$	$l = 10\sqrt{33}$	$x = 9\sqrt{11}$

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B.Ed. NCAS Principal

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- visualise my ultimate success

Robert A Ollis

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