MASTER COACHING

NAME

SINE & COSINE

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Edition 1

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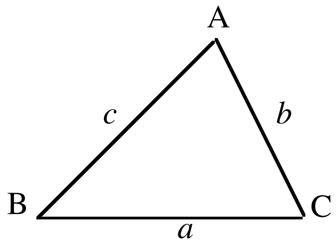
MASTER COACHING

SINE RULE ... by equating areas $\frac{1}{2}a.b\sin C = \frac{1}{2}a.c\sin B = \frac{1}{2}b.c\sin A$



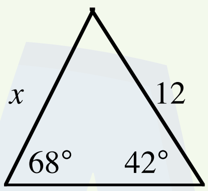
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1 : Find x

Always write down the unknown value first



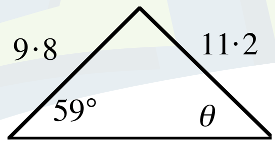
$$\frac{x}{\sin 42°} = \frac{12}{\sin 68°}$$

$$\therefore \quad x = \frac{12\sin 42°}{\sin 68°}$$

$$\approx 8 \cdot 7$$

Example 2 : Find θ

Always write down the unknown value first



$$\frac{\sin\theta}{9 \cdot 8} = \frac{\sin 59°}{11 \cdot 2}$$

$$\therefore \quad \sin\theta = \frac{9 \cdot 8 \sin 59°}{11 \cdot 2}$$

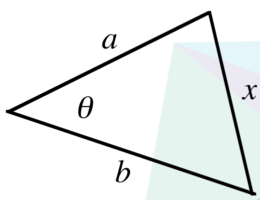
$$\theta \approx 48° \, 36'$$

COSINE RULE

Use this rule when the given data includes

i two sides and the included angle only (SAS),

or **ii** the three sides only (SSS)



$$x^2 = a^2 + b^2 - 2ab\cos\theta \quad \text{or} \quad \cos\theta = \frac{a^2 + b^2 - x^2}{2ab}$$

Trig Formulae Set 2

1 Simplify these trigonometric expressions :

a $\dfrac{1}{\sin^2\theta}$ **b** $\dfrac{\cos^2\theta}{\sin^2\theta}$ **c** $\dfrac{1}{\sec^2\theta}+\dfrac{1}{\operatorname{cosec}^2\theta}$

d $\dfrac{1}{\sin^2\theta}-\dfrac{\cos^2\theta}{\sin^2\theta}$ **e** $\dfrac{1-\sin^2\alpha}{\cos^2\alpha}$ **f** $\tan A.\operatorname{cosec} A$

g $\left(1-\cos^2\theta\right)\left(1+\cot^2\theta\right)$ **h** $\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{\sin\alpha}$ **i** $\dfrac{\cot A}{\cos A}$

j $(1-\sin\theta)(1+\sin\theta)$ **k** $\sin^3\alpha+\cos^2\alpha\sin\alpha$ **l** $\dfrac{\cos\theta}{\operatorname{cosec}\theta}+\dfrac{\sin\theta}{\sec\theta}$

m $1-\dfrac{\sin\theta}{\operatorname{cosec}\theta}$ **n** $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$ **o** $\sqrt{\left(\operatorname{cosec}^2\theta-1\right)\left(1+\tan^2\theta\right)}$

2 Prove these identities : **a** $1+\cos^2\theta=2-\sin^2\theta$ **b** $\sin\theta\tan\theta=\dfrac{1-\cos^2\theta}{\cos\theta}$

c $\sin^4\theta-\cos^4\theta=2\cos^2\theta-1$ **d** $\dfrac{1}{\cot^2\theta}+1=\sec^2\theta$

3 If $\tan A=\frac{3}{4}$ & $\tan B=\frac{1}{2}$ find : **a** $\sin^2 A+\cos^2 A$ **b** $\sin A\cos B+\cos A\sin B$

4 Use the formula $\tan(x-y)^\circ=\dfrac{\tan x^\circ-\tan y^\circ}{1+\tan x^\circ\tan y^\circ}$ to show that $\tan 15^\circ=2-\sqrt{3}$

hint : construct exact value triangles for 30° and 45°

5 If $x=2\sec\theta$ and $y=2\tan\theta$, express x^2-y^2 in its simplest form.

6 Solve for $0\le\theta\le 360^\circ$: **a** $\cos\theta=\cdot 25$ **b** $\cos\theta=-\cdot 25$ **c** $\tan\theta=5$
d $\tan\theta=-5$ **e** $\sin\theta=\cdot 47$ **f** $\sin\theta=-\cdot 47$

7 Solve for $0\le\theta\le 360^\circ$: **a** $\tan\theta=3\sqrt{2}$ **b** $\sin\theta=\frac{1}{2}$ **c** $\cos\theta=0\cdot 783$

d $\tan\theta=-2\cdot 25$ **e** $\sec\theta=\dfrac{2}{\sqrt{2}-1}$ **f** $\sin\theta=\dfrac{-1}{\sqrt{3}+1}$

g $\sin\theta=1\cdot 3$ **h** $1-\cos\theta=0\cdot 426$ **i** $\tan\theta-\sqrt{3}=-2$

j $\sin\theta=-\cos\theta$ **k** $(\tan\theta-1)^2=2$ **l** $\cot\theta=0$

m $1-\cos\theta=1\cdot 328$ **n** $3\sin\theta=2\cos\theta$ **o** $\sec\theta=\sqrt{7}$

Set 2

1 Simplified : **a** $\operatorname{cosec}^2\theta$ **b** $\cot^2\theta$ **c** $\cos^2\theta+\sin^2\theta=1$

d 1 **e** 1 **f** $\sec A$

g 1 **h** $2\sec 2A$ **i** $\operatorname{cosec}\theta$

j $\cos^2\theta$ **k** $\sin\alpha$ **l** $\sin 2\theta$

m $\cos^2\theta$ **n** $2\sec^2\theta$ **o** $\operatorname{cosec}\theta$

2 **a** $1+\cos^2\theta=2-\sin^2\theta$ **b** $\sin\theta\tan\theta=\dfrac{1-\cos^2\theta}{\cos\theta}$

c $\sin^4\theta-\cos^4\theta=2\cos^2\theta-1$ **d** $\dfrac{1}{\cot^2\theta}+1=\sec^2\theta$

3 **a** 1 **b** $\frac{3}{5}\times\frac{2}{\sqrt{5}}+\frac{4}{5}\times\frac{1}{\sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2}{5}\sqrt{5}$

4 $\tan 15^\circ=\tan(45^\circ-30^\circ)=\dfrac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ.\tan 30^\circ}=\dfrac{1-\frac{1}{\sqrt{3}}}{1+1.\frac{1}{\sqrt{3}}}=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$

5 $x^2-y^2=4$

6 **a** $\theta=75^\circ31'$ or $284^\circ29'$ **b** $104^\circ29'$ or $255^\circ31'$ **c** $78^\circ41'$ or $258^\circ41'$

d $101^\circ19'$ or $281^\circ19'$ **f** $28^\circ02'$ or $151^\circ58'$ **g** $208^\circ02'$ or $331^\circ58'$

7 **a** $\theta=76^\circ44',\ 256^\circ44'$ **b** $\theta=30^\circ,\ 150^\circ$ **c** $\theta=38^\circ28',\ 321^\circ52'$

d $\theta=113^\circ57',\ 293^\circ57'$ **e** $\theta=78^\circ3',\ 281^\circ57'$ **f** $\theta=201^\circ28',\ 338^\circ32'$

g No solution **h** $\theta=54^\circ58',\ 305^\circ2'$ **i** $\theta=165^\circ,\ 345^\circ$

j $\theta=135^\circ,\ 315^\circ$ **k** $\theta=67^\circ30',\ 247^\circ30',\ 157^\circ30',\ 337^\circ30'$ **l** $\theta=90^\circ,\ 270^\circ$

m $\theta=109^\circ9',\ 250^\circ51'$ **n** $\theta=33^\circ41',\ 213^\circ41'$ **o** $\theta=67^\circ47',\ 292^\circ13'$