

MASTER COACHING

NAME

NUMBER PLANES

ROB OLLIS

Edition 1

Number Plane

Number plane is geometry, with visual properties made clear by accurate graphs.

The x axis should be drawn on a line of the page.

The scale along the y axis should also fall on the lines of the page.

These two ideas are essential to drawing accurate graphs for most students.

The scale along the x axis could be marked in centimetres.

If the same scale is required on both axes, tear a small strip of paper from the side of the page, rotate it through 90 degrees, and use the line markings to set the scale along the x axis at line widths.

Plotting points : a 'point' is a *location* on a plane, indicated by a capital letter.

The point P which is x units sideways and has a height of y has its location written as $P(x, y)$

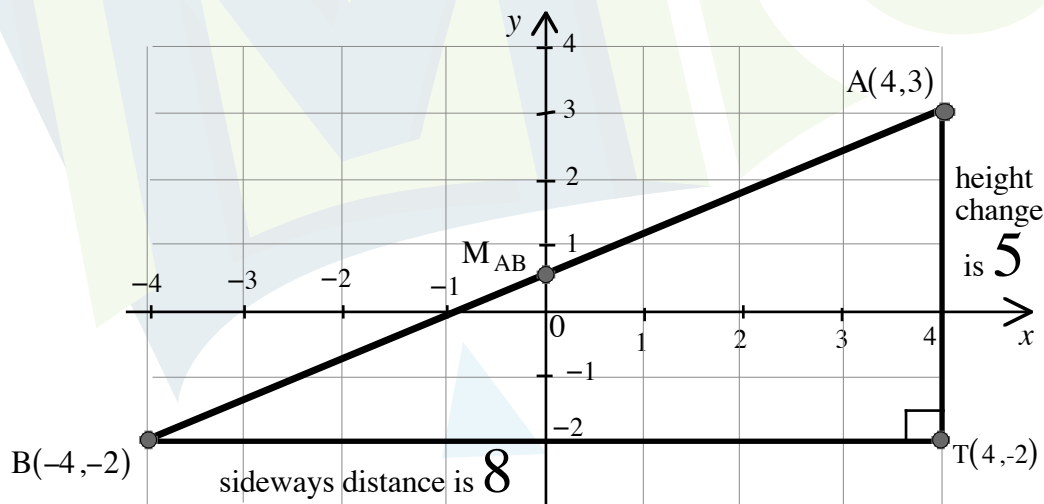
eg $Q(3,7)$ indicates that Q is 3 units to the *positive righthand* side of the vertical y axis and then the positive height of 7 units is above the horizontal x axis

ie (x, y) means $(\text{sideways}, \text{height})$

We must be able to find the length, slope and midpoint of a line joining *two points*.

The equation of the line is not as obvious from the diagram ... see page 4.

Two points ... consider $A(4,3)$ and $B(-4,-2)$ shown plotted on the number plane below :



In the diagram above, I have labeled a point T . Check that its coordinates are $T(4, -2)$

You can see that you have a right-angled triangle ABT , good for Pythagoras.

You need to understand how to get the horizontal length BT and the vertical length AT .

BT is the distance when you go from 4 units to the left of the origin to 4 units on the right.

You can think ' $BT=4+4=8$ ' or you can think ' $BT = 4 - (-4) = 4 + 4 = 8$ '

AT is the distance when you go from 2 units 'below ground' to 3 units 'above ground'.

You can think ' $AT=2+3=5$ ' or you can think ' $AT = 3 - (-2) = 3 + 2 = 5$ '

Length, slope and midpoint depend upon getting these two lengths correct.

Number Plane ... Exercise Set 1 ... Two-Point Measurements

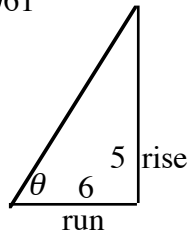
Answers

1 a $AB = \sqrt{5^2 + 6^2} = \sqrt{61}$

b $m_{AB} = \frac{5}{6}$

c $6y = 5x + 8$

d $M_{AB} = \left(-1, \frac{1}{2}\right)$



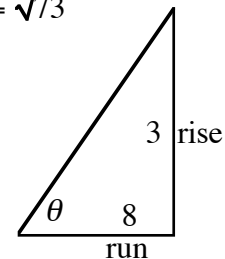
e $y = 1\frac{1}{3}$ **f** $\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39 \cdot 8^\circ$

2 a $AB = \sqrt{8^2 + 3^2} = \sqrt{73}$

b $m_{AB} = \frac{3}{8}$

c $8y = 3x + 17$

d $M_{AB} = \left(1, 2\frac{1}{2}\right)$



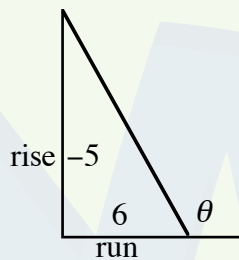
e $y = 2\frac{1}{8}$ **f** $\theta = \tan^{-1}\left(\frac{3}{8}\right) = 20 \cdot 6^\circ$

3 a $AB = \sqrt{5^2 + 6^2} = \sqrt{61}$

b $m_{AB} = \frac{-5}{6}$

c $6y = -5x + 16$

d $M_{AB} = \left(-1, 3\frac{1}{2}\right)$



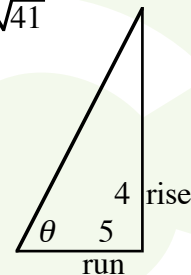
e $y = 2\frac{2}{3}$ **f** $\theta = 140 \cdot 2^\circ$

4 a $AB = \sqrt{4^2 + 5^2} = \sqrt{41}$

b $m_{AB} = \frac{4}{5}$

c $5y = 4x - 19$

d $M_{AB} = \left(3\frac{1}{2}, -1\right)$



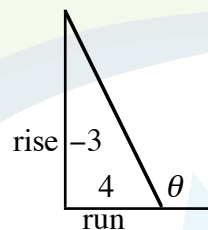
e $y = -3\frac{4}{5}$ **f** $\theta = 38 \cdot 7^\circ$

5 a $AB = \sqrt{3^2 + 4^2} = 5$

b $m_{AB} = \frac{-3}{4}$

c $4y = -3x + 19$

d $M_{AB} = \left(3, 2\frac{1}{2}\right)$



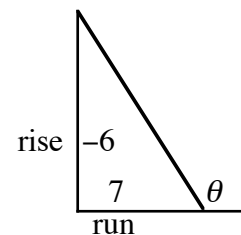
e $y = 4\frac{3}{4}$ **f** $\theta = 143 \cdot 1^\circ$

6 a $AB = \sqrt{6^2 + 7^2} = \sqrt{85}$

b $m_{AB} = -\frac{6}{7}$

c $7y = -6x + 16$

d $M_{AB} = \left(1\frac{1}{2}, 1\right)$



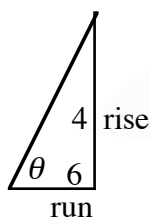
e $y = 2\frac{2}{7}$ **f** $\theta = 139 \cdot 4^\circ$

7 a $AB = \sqrt{4^2 + 6^2} = \sqrt{52}$

b $m_{AB} = \frac{4}{6} = \frac{2}{3}$

c $3y = 2x - 11$

d $M_{AB} = (4, -1)$



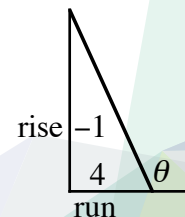
e $y = -3\frac{2}{3}$ **f** $\theta = 33 \cdot 7^\circ$

8 a $AB = \sqrt{1^2 + 4^2} = \sqrt{17}$

b $m_{AB} = -\frac{1}{4}$

c $4y = -x + 11$

d $M_{AB} = (1, 2 \cdot 5)$



e $y = 2\frac{3}{4}$ **f** $\theta = 166 \cdot 0^\circ$

1 Point and Gradient ... find the equation of the line through :

- | | |
|--|--|
| a A(7,3) with gradient 4 | b B(8,-9) with gradient 10 |
| c C(4,17) with $m = 3$ | d D(100,-3) with gradient -5 |
| e E(6,-15) with gradient $-\frac{2}{3}$ | f F(-46,32) with $m = -\frac{1}{2}$ |

2 Parallel ... find the equation of the line through :

- | | |
|---|--|
| a A(2,9) parallel to $4x - 3y + 11 = 0$ | b B(4,-2) // $y = 3x - 978$ |
| c C(-2,7) // $10y - 3x - 6 = 0$ | d D(8,-3) parallel to $3y + 5x = \pi$ |
| e E(12,0) parallel to $6x + 55y + 1 = 0$ | f F(5,-2) // $5x = 7y + 43$ |

3 Mixed ... find the equation of the line through :

- | | |
|---|---|
| a A(8,5) with gradient 3 | b B(12,-5) with gradient -2 |
| c (-3,5) and (2,35) | d D(21,-28) and W(24,2) |
| e E(9,2) parallel to $3x - 10y + 1 = 0$ | f F(8,999) parallel to $y = 101x - 2$ |
| g G(-4,5) perp. to $3x - 10y + 77 = 0$ | h H(8,-3) perpendicular to $y = 7x + 43$ |
| i I(11,0) // to $7y + 4x = 50$ | j J(6,-7) \perp $9y - 4x - 21 = 0$ |
| k K(0,77) parallel to $33x - 2y + 56 = 0$ | l L(5,-9) parallel to $y = 7x - 333$ |
| m M(4,-5) perp. to $x - y + 1 = 0$ | n N(-1,-2) perpendicular to $y = 3x + 4$ |
| o (99,1) // to $3x + 5y + 99 = 0$ | p P(3,9) \perp $5y + 7x - 8 = 0$ |
| q Q(10,41) parallel to $13x - 3y + 71 = 0$ | r R(30,-21) parallel to $y = 4x + 1109$ |

Answers:

1 Point and Gradient

- | | |
|-----------------------------------|----------------------------------|
| a $y = 4x - 25$ | b $y = 10x - 89$ |
| c $y = 3x + 5$ | d $y = -5x + 497$ |
| e $y = -\frac{2}{3}x - 11$ | f $y = -\frac{1}{2}x + 9$ |

2 parallel

- | | |
|------------------------------|-------------------------|
| a $4x - 3y + 19 = 0$ | b $y = 3x - 14$ |
| c $10y - 3x - 76 = 0$ | d $3y + 5x = 31$ |
| e $6x + 55y - 72 = 0$ | f $5x = 7y + 39$ |

3 Mixed

- | | | |
|------------------------------|-------------------------------|------------------------------|
| a $y = 3x - 19$ | b $y = -2x + 19$ | c $y = 6x + 23$ |
| d $y = 10x - 238$ | e $3x - 10y - 7 = 0$ | f $y = 101x + 191$ |
| g $10x + 3y + 25 = 0$ | h $7y = -x - 13$ | i $7y + 4x = 44$ |
| j $4y + 9x - 26 = 0$ | k $33x - 2y + 154 = 0$ | l $y = 7x - 44$ |
| m $x + y + 1 = 0$ | n $3y = -x - 7$ | o $3x + 5y - 302 = 0$ |
| p $7y - 5x - 48 = 0$ | q $13x - 3y - 7 = 0$ | r $y = 4x - 141$ |