

MASTER COACHING

NAME

CALCULUS OF FUNCTIONS

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Edition 1

Calculus.

Differentiating Powers of x

To differentiate $y = 5x^6$ $P = 6$ (P stands for the power of x)

the process of differentiation involves multiplying the power of x by the coefficient and the lowering the power by one.

For example

$$\begin{aligned}
 y &= 5x^6 & P &= 6 - 1 \\
 \frac{dy}{dx} &= 6 \times 5 x^{6-1} & &= 5 \\
 &= 30 x^5 & &
 \end{aligned}$$

Further Examples

$$\begin{aligned}
 y &= \frac{7}{5x^3} & P &= -3 - 1 & \text{or} & y = 8x^3 \sqrt{x} & P &= 3\frac{1}{2} - 1 \\
 \frac{dy}{dx} &= \frac{7 \times -3}{5x^4} & &= -4 & & \frac{dy}{dx} &= 28x^2 \sqrt{x} & &= 2\frac{1}{2} \\
 &= \frac{-21}{5x^4} & & & & & & &
 \end{aligned}$$

Please note that the answer to the question is given in exactly the same format as the original question and also the format of the question does not have to be altered to answer the question. It is very important at this stage to write down $P =$ each time you do these questions because once the power of the variable has been identified the solution to the problem is routine. Note also, if there is a fractional part to the index then this part doesn't change unless $0 < P < 1$

Once you can differentiate single powers of x the process of differentiating function of functions can be simplified by remembering three simple rules and using the standard integral sheet.

The three rules are

1. Differentiate the prime function first

2 Don't change anything inside the bracket. This is the source of most students mistakes.

3. After differentiating the prime function multiply by the differential of the bracket.

The teacher should then give a few examples such as :-differentiate with respect to

$$\begin{array}{lll}
 (1) \sin(x^2 + x) & (2) 3\cos(3x - 2x^2) & (3) 4\ln(5x^2 + 3x) \\
 (4) 8\ln(\sin(x)) & (5) 3\tan(5x - 3) & (6) 6e^{4x^2 - x} \\
 (7) 5\sec(e^x) & (8) 3\tan^{-1}(2x^2) & \text{etc.}
 \end{array}$$

and make the students use the standard integral sheet to differentiate the prime function. The above examples can be given to a Advanced maths group even though $\tan^{-1}(x)$ is not in their course.

Once differentiation has been mastered then integration should follow easily.

For simple index types the rule is

1. Raise the power by one

2. divide by the new power.

Again the secret is to write $P =$. Consider the following example, variations of which can be generated at will in clusters of 5 or 6 quick questions to generate speed and accuracy in the group.

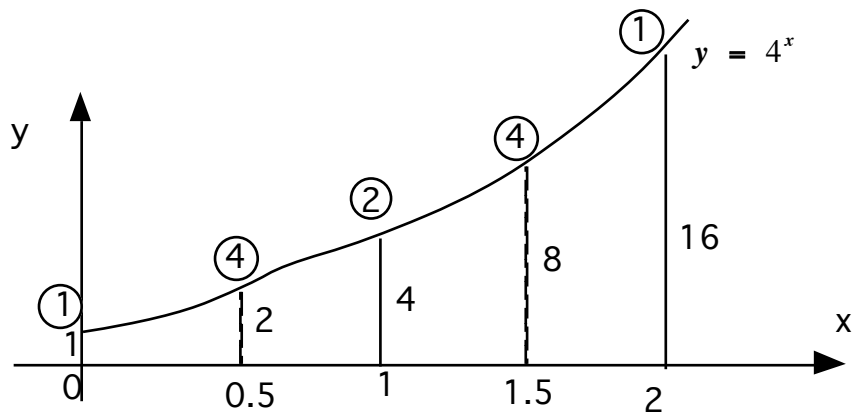
$$\int 5x^3 - 8x^2\sqrt{x} + \frac{3}{4x^2} dx = \frac{5x^4}{4} - \frac{8x^3\sqrt{x}}{3\frac{1}{2}} + \frac{3}{4x \times -1} + c$$

$$\begin{array}{lll}
 P = 3+1 & P = 2\frac{1}{2}+1 & P = -2+1 \\
 = 4 & = 3\frac{1}{2} & = -1
 \end{array}
 \quad = \frac{5x^4}{4} - \frac{16x^3\sqrt{x}}{7} - \frac{3}{4x} + c$$

or

$$\int \frac{7}{4x^3\sqrt{x}} - \frac{5}{3\sqrt{x}} + \frac{7x^2}{5} dx = \frac{7}{4x^2\sqrt{x} \times -2\frac{1}{2}} - \frac{5\sqrt{x}}{3 \times \frac{1}{2}} + \frac{7x^3}{5 \times 3} + c$$

$$\begin{array}{lll}
 P = -3\frac{1}{2}+1 & P = -\frac{1}{2}+1 & P = 2+1 \\
 = -2\frac{1}{2} & = \frac{1}{2} & = 3
 \end{array}
 \quad = \frac{-7}{10x^2\sqrt{x}} - \frac{10\sqrt{x}}{3} + \frac{7x^3}{15} + c$$



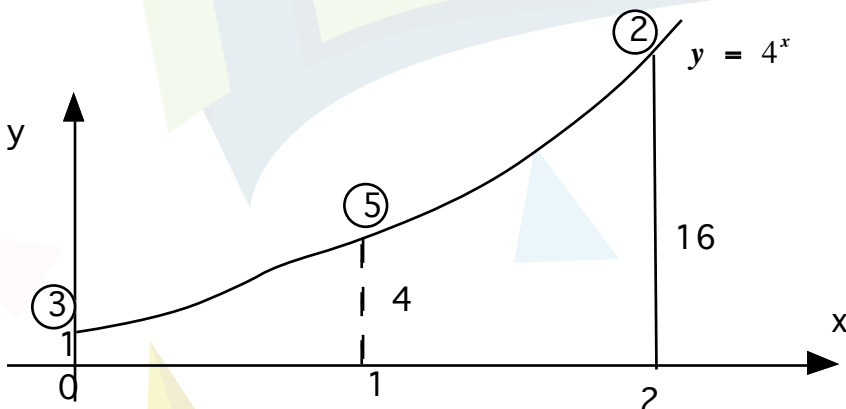
Base = 1 unit

Note: $x = 1$ is an end side of the first strip and a beginning side of the second strip, so it is used twice

$$\begin{aligned} \text{Average height} &\approx \frac{1 \times 1 + 4 \times 2 + 2 \times 4 + 4 \times 8 + 1 \times 16}{6} & \therefore \int_0^2 4^x dx &\approx 1 \times 10 \frac{5}{6} \\ &\approx 10 \frac{5}{6} & &\approx 10 \frac{5}{6} \end{aligned}$$

Students may wish to devise their own rule. Any weighted mean of heights will give a genuine approximation to the area under a curve, even if it is not as good as Simpson's Rule.

eg Bob's Rule. (or your rule)



Base = 2 units

$$\begin{aligned} \text{Average height} &\approx \frac{3 \times 1 + 5 \times 4 + 2 \times 16}{10} & \therefore \int_0^2 4^x dx &\approx 2 \times 5 \frac{1}{2} \\ &\approx 5 \frac{1}{2} & &\approx 11 \end{aligned}$$

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1. Smile, relax, you are focused and in control
2. Concentrate, visualize, execute: claim the reward
3. Assume success. Our members should approach each test in life enthusiastically; every challenge presents an opportunity to demonstrate your prowess. Relish but don't underestimate the magnitude of the test, instead focus your thoughts towards a positive outcome, a chance to excel; a time to enjoy your moment in the sun.

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- show care and give encouragement to you in your striving for excellence
- to personalise all our efforts to your specific needs in all areas
- to encourage you to dare to dream, and to expect that dreams do come true

Christian Avent

CHRISTIAN AVENT
B.Ed. NCAS Principal

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Robert A Ollis

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Founder, Master Coaching

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